MATH 521A: Abstract Algebra Exam 3

Please read the following instructions. For the following exam you are free to use a calculator and any papers you like, but no books or computers. Please turn in **exactly four** problems. You must do problems 1-3, and one more chosen from 4-6. Please write your answers on separate paper, make clear what work goes with which problem, and put your name or initials on every page. You have 50 minutes. Each problem will be graded on a 5-10 scale (as your quizzes), for a total score between 20 and 40. This will then be multiplied by $\frac{5}{2}$ for your exam score.

Turn in problems 1,2,3:

1. Set $f(x) = 3x^3 + 5x^2 + 6x$, $g(x) = 3x^4 + 5x^3 + x^2 + 3x + 2$, both in $\mathbb{Z}_7[x]$. Use the extended Euclidean algorithm to find $\gcd(f, g)$ and to find polynomials $s(x), t(x)$ such that $\gcd(f, g) = f(x)s(x) + g(x)t(x)$.

2. Factor $f(x) = x^4 + x^3 + 6x^2 - 14x + 16 \in \mathbb{Q}[x]$ into irreducibles.

3. Let F be a field. We define the "derivative" operator $D : F[x] \to F[x]$ via

   $$D(a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0) = na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + 1a_1.$$

   This operator satisfies, for all $f, g \in F[x]$ and for all $c \in F$:
   (a) $D(f + g) = D(f) + D(g)$; (b) $D(cf) = cD(f)$; (c) $D(fg) = fD(g) + D(f)g$

   Suppose $f, g \in F[x]$ and $f^2|g$. Prove that $f|D(g)$.

Turn in exactly one more problem of your choice:

4. Set $f(x) = x + 2x^2, g(x) = x + 4x^2$, both in $\mathbb{Z}_8[x]$. Prove that $f|g$ and $g|f$.

5. Set $f(x) = x^n + x^{n-1} \in F[x]$. Carefully determine all divisors of $f(x)$.

6. For ring R, $a \in R$, and $n \in \mathbb{N}$, we say a has *additive order* n if $\underbrace{a + a + \cdots + a}_{n} = 0_R$, and for $m < n$ we have $\underbrace{a + a + \cdots + a}_{m} \neq 0_R$. We write this $ord_R(a) = n$. Suppose every element of R has an order (not necessarily the same one). Prove that every element of $R[x]$ has an order.

You may also turn in the following (optional):

7. Describe your preferences for your final group assignment. (will be kept confidential)